

Oscillator Strengths for the Helium Isoelectronic Sequence*

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Variational wavefunctions have been computed for the 1^1S , $2^1,^3S$, $2^1,^3P$, $3^1,^3P$, and $3^1,^3D$ terms of ions in the helium isoelectronic sequence from He through Ne^{+8} . The trial functions are Hylleraas type expansions, explicitly involving r_{12} , and with expansion lengths of around 50 terms, differing slightly for the different states. Oscillator strengths have been computed for all the allowed transitions by both the dipole length and velocity formulas. Agreement between these forms, comparisons with very accurate data for neutral helium, and the accuracy of the computed energies all point to considerable reliability of the results for the entire sequence, probably around 5 percent, if not better.

Key Words: Atomic term value, oscillator strengths, wavefunctions.

1. Introduction

While the spectra of neutral helium and singly ionized lithium have been studied quite thoroughly [1, 2],¹ the data available for the rest of this isoelectronic sequence leaves much to be desired. Be III, for instance, is the last ion for which the 2^1S level has even been observed, and it is common for only a dozen or so levels to be known for any given ion [3]. The situation for oscillator strengths, of course, is no better, and with the exception of helium, where some accurate calculations have recently been made [4, 5], there is practically no reliable f -value data available. With the observational possibilities opened up by high energy plasma devices, accelerators, and astronomical spectroscopy in the ultraviolet, this gap in basic atomic data could assume considerable importance.

On the other hand, two-electron atomic systems are well-known for the relative ease with which one can make accurate variational calculations of wavefunctions and energies. The pioneering work of Hylleraas [6] showed that a linear expansion trial function with terms explicitly containing the interelectronic coordinate, r_{12} , would converge rapidly, and that the matrix elements could be given in a closed form amenable to numerical computation. Since then, helium, and to a lesser extent, the helium-like ions have been a favorite target for accurate variational calculations, primarily on the ground state. The most definitive calculations have recently been made by Pekeris and co-workers [7], who have also done extensive calculations on the helium excited states [8]; they have also

done some work on the Li^+ excited states [9]. Recently, also, configuration interaction expansions, which do not explicitly involve r_{12} , have yielded good results for the helium term values [10] and oscillator strengths [5].

In view of these observations, it seems clear that a systematic calculation of oscillator strengths for the isoelectronic sequence is not only desirable, but long overdue. With electronic computers, it is almost trivial to make calculations of relatively modest expansion lengths, which should still be sufficient to yield fairly definitive results. This paper reports the results of such calculations, employing of the order of 50 terms, for transitions among the 1^1S , $2^1,^3S$, $2^1,^3P$, $3^1,^3P$, and $3^1,^3D$ terms of the helium sequence through Ne^{+8} .

2. Description of the Calculations

The Rayleigh-Ritz variational procedure is employed to calculate energies and wavefunctions, using the usual non-relativistic, spin-independent Hamiltonian, which in atomic units,² is given by,

$$\mathcal{H} = \frac{\Delta_1}{2} - \frac{\Delta_2}{2} - Z/r_1 - Z/r_2 + \frac{1}{r_{12}} \quad (1)$$

It is clear that all magnetic splittings and relativistic shifts are ignored, which can have a noticeable effect on the energy levels of the higher ions. The trial functions are taken to be linear expansions of the form,

$$\psi = \sum_n c_n \Phi_n \quad (2)$$

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¹ Figures in brackets indicate the literature references at the end of this paper.

² In atomic units, $m_e = \hbar = 1$. The unit of length is the Bohr radius, 0.52917 Å, and the energy unit is $2R_M$, where R_M is the appropriate reduced mass Rydberg.

The c_n are linear variational parameters, and the two-electron basis functions, Φ_n , are of the usual Hylleraas variety,

$$\Phi_{pq\mu} = e^{-\rho_1 - x\rho_2} \rho_1^p \rho_2^q \rho_{12}^\mu Y_l^m(\theta_2, \varphi_2) \pm e^{-\rho_2 - x\rho_1} \rho_2^p \rho_1^q \rho_{12}^\mu Y_l^m(\theta_1, \varphi_1) \quad (3)$$

with $\rho = \zeta r$, and where ζ and x are additional nonlinear parameters, Y_l^m is the spherical harmonic appropriate for the desired angular momentum state, and the + and - signs refer to the singlet and triplet states, respectively. Since the spin functions factor out in the two-electron problem, all spin dependence has been suppressed.

Requiring the variation of the total energy,

$$E = \frac{\langle \psi | \mathcal{H} | \psi \rangle}{\langle \psi | \psi \rangle} \quad (4)$$

to be stationary with respect to variations of the coefficients leads to the matrix eigenvalue equation.

$$\sum_j H_{ij} C_j = E \sum_j S_{ij} C_j \quad (5)$$

$$H_{ij} = \langle \Phi_i | \mathcal{H} | \Phi_j \rangle, S_{ij} = \langle \Phi_i | \Phi_j \rangle. \quad (6)$$

By selecting the n th eigenvalue, one has an upper bound to the energy of the corresponding excited state and, with appropriate nonlinear parameters in (3), an approximation to its wavefunction [11].

The oscillator strength for an electric dipole transition is given by

$$f = \frac{2}{3} \frac{\Delta E_{ij}}{g_i} |\langle \psi_i | \mathbf{r} | \psi_j \rangle|^2. \quad (7)$$

where i and j refer to the initial and final states respectively, ΔE_{ij} is the energy difference in atomic units, and g_i is the statistical weight of the initial state. It is further supposed that the squared matrix element has been summed over any degeneracies in either or both states. An alternative form is the dipole velocity form of the oscillator strength,

$$f^v = \frac{2}{3} \frac{1}{g_i \Delta E_{ij}} |\langle \psi_i | \nabla | \psi_j \rangle|^2 \quad (8)$$

which should be identical in value with the dipole length form, although for approximate wavefunctions it may, and often does, disagree.

For sake of future reference, some of the other standard approximation schemes should probably be sketched here. The configuration interaction method follows the same general procedure outlined above, choosing the trial function to be a linear expansion as in (2). The two-electron basis, however, consists of products of one-electron functions (orbitals), of the general form

$$\Phi_{pq} = \varphi_p(1)\varphi_q(2) \pm \varphi_p(2)\varphi_q(1). \quad (3')$$

The Hartree-Fock model follows by choosing the trial function to be only the first term of a configuration interaction expansion with the orbitals occupied according to the *aufbau* principle, and adjusting the form of the orbitals to minimize the total energy.

3. Results and Discussion

The total energies computed for neutral helium are given in table 1 and compared with some other calculations. With the exception of the $4^1, 3^1\text{P}$ levels, all of the present calculations used expansion lengths of 52–54 terms. Positive powers of r_1 , r_2 , and r_{12} through degree 5 were used, but with some selection, omitting some of the higher degree terms which made little difference in the total energy. For the $4p$ levels, expansions of only 25 terms were used, because of difficulties encountered in getting the diagonalization to converge on the appropriate root for the longer expansions. It can be seen that, with the possible exception of the ground state, the configuration interaction calculation of Green et al. [5], is quite comparable to the ones reported here, and both compare quite well with the still more accurate calculations. The sole exception to this might be the $4p$ levels calculated here, and for the reasons indicated above.

Tables 2 and 3 contain a similar comparison of oscillator strength calculations for neutral helium, including, in addition, the results of the Coulomb approximation of Bates and Damgaard [12]. The Hartree-Fock values reported here are not quite the same as in Trefftz et al. [13], due to the fact that the exact energy differences ΔE , were used in eqs (7) and (8). This was done throughout this paper, since it throws the burden of any discrepancies entirely on the calculated transition moment, which is the theoretically more critical quantity.

Probably the most obvious comment one can make about these results is that all the approximations dis-

TABLE 1. Computed total energies (in a.u.) for neutral helium

Term	Hartree-Fock ^a	Configuration interaction ^b	This paper	Accurate ^c
1 ¹ S	-2.86168	-2.90338	-2.90372	-2.90372
2 ¹ S	-2.14347	-2.14594	-2.14597	-2.14597
3 ¹ S		-2.06126	-2.06126	-2.06127
2 ³ P	-2.12246	-2.12378	-2.12384	-2.12384
3 ³ P	-2.05474	-2.05513	-2.05513	-2.05515
4 ¹ P	-2.03090	-2.03106	-2.03095	-2.03107
3 ¹ D	-2.05555	-2.05561	-2.05562	(-2.05562)
2 ³ S	-2.17425	-2.17521	-2.17523	-2.17523
3 ³ S	-2.06849	-2.06868	-2.06869	-2.06869
2 ³ P	-2.13143	-2.13313	-2.13316	-2.13316
3 ³ P	-2.05750	-2.05807	-2.05807	-2.05808
4 ³ P	-2.03213	-2.03232	-2.03220	-2.03232
3 ³ D	-2.05557	-2.05563	-2.05564	(-2.05564)

^a See reference [13].

^b See reference [10].

^c Except for the 3D terms, see references [8] and [9]. These terms were obtained from the observed levels—reference [2].

TABLE 2. Comparison of oscillator strength calculations for neutral helium—the singlet transitions

Transition	$\Delta E(\text{a.u.})$	Type	Coulomb approx. ^a	Hartree-Fock ^b	Configuration interaction ^c	This paper	Accuracy ^d
1 ¹ S → 2 ¹ P	0.77988	len. vel.	0.259	0.258 .239	0.2754 .2759	0.2759 .2761	0.2762 .2762
1 ¹ S → 3 ¹ P	.84858	len. vel.	.0713	.0707 .0646	.0729 .0730	.0734 .0730	.0736 .0734
1 ¹ S → 4 ¹ P	.87265	len. vel.	.0312	.0312 .0266	.0296 .0296	.0302 .0303	
2 ¹ S → 2 ¹ P	0.02213	len. vel.	0.390	0.390 .340	0.3773 .3950	0.3764 .3774	0.3764 .3764
2 ¹ S → 3 ¹ P	.09083	len. vel.	.160	.160 .154	.1513 .1540	.1478 .1506	.1514 .1514
2 ¹ S → 4 ¹ P	.11490	len. vel.	.051	.0513 .0498	.0493 .0506	.0508 .0506	
3 ¹ S → 2 ¹ P	0.06257	len. vel.			0.1458 .1446	0.1425 .1462	
3 ¹ S → 3 ¹ P	.00612	len. vel.			.628 .645	.625 .634	
3 ¹ S → 4 ¹ P	.03020	len. vel.			.1429 .1433	.134 .145	
2 ¹ P → 3 ¹ D	0.06822	len. vel.	0.738	0.738 .711	0.7106 .7095	0.7064 .7148	
3 ¹ P → 3 ¹ D	.00048	len. vel.	.021	.021 .064	.022 .021	.021 .026	
4 ¹ P → 3 ¹ D	.02454	len. vel.			.0152 .0152	.0142 .0144	

^a See reference [12].

^b See reference [13].

^c See reference [5].

^d See reference [4].

TABLE 3. Comparison of oscillator strength calculations for neutral helium—the triplet transitions

Transition	$\Delta E(\text{a.u.})$	Type	Coulomb approx. ^a	Hartree-Fock ^b	Configuration interaction ^c	This paper	Accuracy ^d
2 ³ S → 2 ³ P	0.04207	len. vel.	0.538	0.558 .611	0.5398 .5487	0.5391 .5401	0.5391 .5391
2 ³ S → 3 ³ P	.11715	len. vel.	.063	.0569 .0503	.0644 .0668	.0641 .0634	.0645 .0645
2 ³ S → 4 ³ P	.14288	len. vel.	.026	.0234 .0204	.0259 .0271	.0240 .0222	
3 ³ S → 2 ³ P	.06446	len. vel.			.209 .206	.207 .209	
3 ³ S → 3 ³ P	.01061	len. vel.			.892 .911	.891 .898	
3 ³ S → 4 ³ P	.03637	len. vel.			.0499 .0491	.0433 .0425	
2 ³ P → 3 ³ D	.07753	len. vel.	.620	.637 .610	.6105 .6269	.6084 .6121	
3 ³ P → 3 ³ D	.00245	len. vel.	.112	.112 .074	.1130 .1216	.1121 .1097	
4 ³ P → 3 ³ D	.02329	len. vel.			.0372 .0378	.0340 .0344	

^a See reference [12].

^b See reference [13].

^c See reference [5].

^d See reference [4].

played here are good. A straight average of the presumably most sophisticated calculations (both length and velocity) of the last three columns provides a value with which any one of these individual calculations agrees to within 5 to 10 percent, and often much better.

Even the Coulomb approximation agrees very well with such a "recommended" value. It is obvious that the helium spectrum is quite Coulombic with the transitions corresponding either to a lone electron jumping around outside a very compact core, or to a single electron jumping out of a very compact ground state to a very diffuse orbital. Clearly, calculations of the degree of sophistication of the r_{12} or configuration interaction approximations amount to using an elephant gun to kill a gnat.

However, the elephant gun will continue to be used for the rest of the isoelectronic sequence. One reason for doing so is the fact that the energy level data needed for the Coulomb approximation rapidly becomes quite sparse along the sequence. Variational calculations such as the present ones generate their own energy levels and, as a byproduct, make some term value predictions of some interest in their own right. Secondly, and most importantly, the degree of "overkill" attained by such calculations, as indicated by neutral He, provides a valuable accuracy cushion for the prediction of these unknown oscillator strengths.

The total energies computed for the isoelectronic sequence through Ne^{+8} are shown in table 4. A comparison with some more accurate values for Li^+ [7, 8, 9], indicates agreement in the total energy always within 1×10^{-5} a.u. The ground state energies agree better than this with the calculations of Pekeris [7] for the entire sequence. Probably the most interesting feature of these calculations is the prediction that the 2¹S and 2³P levels interchange their positions as one moves along the isoelectronic sequence, with the crossing taking place between C^{+4} and N^{+5} . It can also be seen that the 3¹P level remains the highest of the $n=3$ levels for the entire sequence, at least through Ne^{+8} . It is well-known for neutral helium that, among all the levels arising from the same *principal* quantum numbers, e.g., 4^{1,3}S, 4^{1,3}P, . . . , the 1¹P is always the highest, at least through $n=7$. These results suggest that this remains the case for the isoelectronic positive ions, certainly for $n=3$.

It should be emphasized that all of these calculations are purely nonrelativistic and, hence, include none of the shift or splitting due to relativistic or radiative corrections. While these nonrelativistic values are probably quite accurate, such corrections can become large for the higher ions. The point of the present calculations has been to obtain wavefunctions with which to compute the oscillator strengths and not to accurately predict observable energies. Also, although the energy corrections are large, the non-relativistic calculations still give a fairly detailed and accurate representation of the spectrum. This is illustrated by table 5, where the calculated and observed [3] term values of O VII are compared. The relativistic and Lamb shifts in the ground state have been calculated [7] to be 3550 cm^{-1} and -380 cm^{-1} , respectively, which compares well with the error here of 3020 cm^{-1} . The experimental ground state term value is uncertain by 600 cm^{-1} . For Li^+ , the relativistic shifts in the 1¹S, 2¹S, 2³S, 2¹P levels have been calculated [9] as 19.7, 6.5, 16.9, 5.6 cm^{-1} respec-

TABLE 4. Computed total energies, $-E$, (in a.u.) for the helium isoelectronic sequence

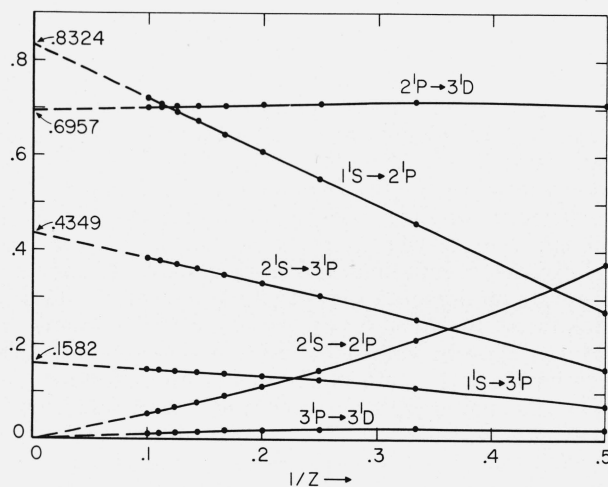
Term	Expansion length	$Z \rightarrow 2$	3	4	5	6	7	8	9	10
1 ¹ S	53	2.90372	7.27991	13.65557	22.03097	32.40624	44.78144	59.15660	75.53171	93.90680
2 ³ S	54	2.17523	5.11073	9.29717	14.73390	21.42076	29.35768	38.54464	48.98164	60.66865
2 ¹ S	54	2.14597	5.04087	9.18487	14.57852	21.22198	29.11540	38.25875	48.65204	60.29530
2 ³ P	52	2.13316	5.02771	9.17497	14.57313	21.22171	29.12050	38.26942	48.66842	60.31748
2 ¹ P	52	2.12384	4.99335	9.11077	14.47728	21.09333	28.95911	38.07473	48.44024	60.05567
3 ³ P	52	2.05806	4.73045	8.51460	13.41003	19.41672	26.53455	34.76352	44.10361	54.55484
3 ³ D	52	2.05564	4.72253	8.50058	13.38977	19.39008	26.50151	34.72405	44.05771	54.50248
3 ¹ D	52	2.05562	4.72237	8.50020	13.38906	19.38901	26.50006	34.72218	44.05542	54.49974
3 ¹ P	52	2.05513	4.72018	8.49594	13.38268	19.38049	26.48943	34.70935	44.04044	54.48266

TABLE 5. Computed and observed term values relative to the ionization limit (in cm^{-1}) for O VII

Term	Computed	Observed ^a	Error
1 ¹ S	5959980	5963000	3020
2 ³ S	1436330	1437730	1400
2 ³ P ₂	1375930	1376140	460 (ave.)
3 ³ P ₁	—	690	—
3 ³ P ₀	—	750	—
2 ¹ S	1373590	—	—
2 ¹ P	1333200	1333800	600
3 ³ P	606500	606700	200
3 ³ D	597840	597930	90
3 ¹ D	597430	—	—
3 ¹ P	594610	594450	160

^a See reference [3].

tively, so the excited state errors here are, at least, not unreasonable. The unobserved 2¹S level is here predicted to lie about 1600 cm^{-1} above the 2³P center of gravity.

FIGURE 1. Singlet oscillator strengths for the helium isoelectronic sequence versus $1/Z$.

The oscillator strengths, finally, are given in tables 6 and 7, and they are plotted as a function of $1/Z$ in figures 1 and 2. As has been pointed out earlier [14], the oscillator strength for an isoelectronic sequence has a descending power of Z expansion of the form

$$f = a_0 + a_1/Z + a_2/Z^2 + \dots \quad (9)$$

where a_0 is just the oscillator strength computed in a purely hydrogenic approximation (if there are no zeroth order degeneracies). Also, a_0 vanishes for transitions involving no change in the principal quantum number. Thus, the infinite Z limit is either the hydrogenic value or zero, depending on whether $n \rightarrow n'$ or $n \rightarrow n$, and it can be seen that all the plotted f -values (mean of length and velocity) behave very nicely in accordance with these limiting situations.

4. Conclusions

In general, it seems reasonable to recommend the mean of the length and velocity values for any given transition of the sequence with an uncertainty of about

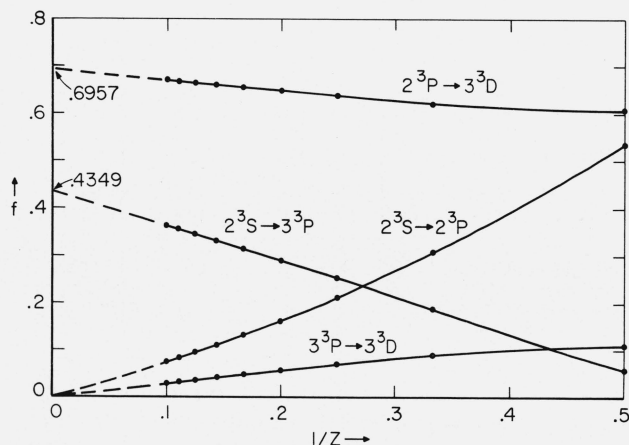
FIGURE 2. Triplet oscillator strengths for the helium isoelectronic sequence versus $1/Z$.

TABLE 6. Oscillator strengths for the helium isoelectronic sequence—singlet transitions

Transition \ Z	2	3	4	5	6	7	8	9	10
$1^1S \rightarrow 2^1P$	<i>v.</i>	0.2761	0.4566	0.5515	0.6089	0.6470	0.6742	0.6944	0.7101
	<i>l.</i>	.2759	.4565	.5515	.6089	.6470	.6742	.6944	.7101
	ΔE	0.77988	2.28657	4.54480	7.55369	11.31291	15.82233	21.08187	27.09147
$1^1S \rightarrow 3^1P$	<i>v.</i>	.0730	0.1103	0.1267	0.1352	0.1404	0.1438	0.1461	0.1478
	<i>l.</i>	.0734	.1107	.1269	.1355	.1406	.1439	.1462	.1479
	ΔE	0.84859	2.55973	5.15963	8.64829	13.02575	18.29201	24.44725	31.49127
$2^1S \rightarrow 2^1P$	<i>v.</i>	.3774	0.2129	0.1487	0.1145	0.0932	0.0786	0.0679	0.0599
	<i>l.</i>	.3764	.2126	.1485	.1144	.0930	.0785	.0679	.0598
	ΔE	0.02213	0.04753	0.07410	0.10124	0.12865	0.15629	0.18402	0.21180
$2^1S \rightarrow 3^1P$	<i>v.</i>	.1506	.2568	.3061	.3340	.3524	.3647	.3736	.3807
	<i>l.</i>	.1478	.2551	.3046	.3326	.3495	.3627	.3728	.3791
	ΔE	0.09083	0.32069	0.68892	1.19584	1.84149	2.62597	3.54940	4.61160
$2^1P \rightarrow 3^1D$	<i>v.</i>	.7148	.7173	.7139	0.7121	0.7096	0.7078	0.7065	0.7050
	<i>l.</i>	.7064	.7108	.7084	.7065	.7046	.7034	.7024	.7016
	ΔE	0.06822	0.27097	0.61057	1.08822	1.70432	2.45906	3.35254	4.38482
$3^1P \rightarrow 3^1D$	<i>v.</i>	.0249	.0288	.0242	0.0221	0.0188	0.0165	0.0147	0.0131
	<i>l.</i>	.0212	.0244	.0211	.0178	.0152	.0132	.0117	.0105
	ΔE	0.00048	0.00219	0.00426	0.00638	0.00852	0.01063	0.01283	0.01498

5 percent. The length-velocity agreement is usually much better than this, and the experience with helium indicates that it should be a safe margin of uncertainty. The sole exception is the 3^1P - 3^1D transition, where the length-velocity disagreement is somewhat larger, and this transition should probably be regarded with some skepticism for the entire sequence, although the length value is probably to be preferred. The energy differences here are small, indicating a very small velocity transition moment, which is thus apt to be unreliable, as suggested by the comparisons of table 2. For the higher members of the sequence, the oscillator

strengths, at least the non-relativistic values, can be read directly off the curves of figures 1 and 2.

For very highly ionized species, intersystem transitions become strong and eventually will begin to steal oscillator strength from the allowed transitions. This probably does not become too serious for the ions computed here. The spin interactions are still rather weak compared to the electrostatic, e.g., the 3P spin splitting in O VII is about 600 cm^{-1} while the 3P - 1P separation is of the order of $40,000 \text{ cm}^{-1}$. However, the precise effect of the intersystem transitions will have to await further calculations.

TABLE 7. Oscillator strengths for the helium isoelectronic sequence—triplet transitions

Transition \ Z										
$2^3S \rightarrow 2^3P$	<i>v.</i>	0.5401	0.3086	0.2135	0.1627	0.1315	0.1102	0.0949	0.0832	0.0742
	<i>l.</i>	.5391	.3080	.2131	.1626	.1314	.1102	.0948	.0833	.0742
	ΔE	0.04207	0.08301	0.12220	0.16077	0.19905	0.23718	0.27522	0.31322	0.35117
$2^3S \rightarrow 3^3P$	<i>v.</i>	.0634	.1857	.2523	.2908	.3163	.3343	.3476	.3576	.3656
	<i>l.</i>	.0641	.1872	.2526	.2910	.3163	.3341	.3470	.3572	.3651
	ΔE	0.11717	0.38028	0.78157	1.32387	2.00404	2.82313	3.78112	4.87803	6.11381
$2^3P \rightarrow 3^3D$	<i>v.</i>	.6121	.6263	.6404	.6504	.6575	.6626	.6667	.6696	0.6723
	<i>l.</i>	.6084	.6243	.6390	.6493	.6565	.6618	.6659	.6691	.6717
	ΔE	0.07753	0.30519	0.67439	1.18336	1.83163	2.61899	3.54537	4.61071	5.81500
$3^3P \rightarrow 3^3D$	<i>v.</i>	.1097	.0901	.0712	.0578	.0485	.0420	.0368	.0328	.0294
	<i>l.</i>	.1121	.0906	.0711	.0577	.0485	.0417	.0365	.0325	.0293
	ΔE	.00243	.00792	.01402	.02026	.02664	.03304	.03947	.04590	.05236

5. References

- [1] G. Herzberg and H. R. Moore, Can. J. Phys. **37**, 1293 (1959).
- [2] W. C. Martin, J. Res. NBS **64A**, 19 (1960).
- [3] B. Edlen, Arkiv f. Fysik **4**, 441 (1951).
- [4] B. Schiff and C. L. Pekeris, Phys. Rev. **134**, A638 (1964).
- [5] L. C. Green, N. C. Johnson and E. K. Kolchin, Astrophys. J. **144**, 369 (1966).
- [6] E. A. Hylleraas, Z. Physik **54**, 347 (1929).
- [7] C. L. Pekeris, Phys. Rev. **112**, 1649 (1958); Phys. Rev. **115**, 1216 (1959).
- [8] C. L. Pekeris, Phys. Rev. **127**, 509 (1962); C. L. Pekeris, B. Schiff, and H. Lifson, Phys. Rev. **126**, 1057 (1962).
- [9] C. L. Pekeris, Phys. Rev. **126**, 143 (1962); B. Schiff, H. Lifson, C. L. Pekeris, and P. Rabinowitz, Phys. Rev. **140**, A1104, (1965).
- [10] L. C. Green, E. K. Kolchin, and N. C. Johnson, Phys. Rev. **139**, A373 (1965).
- [11] J. K. L. MacDonald, Phys. Rev. **43**, 830 (1933).
- [12] D. R. Bates and A. Damgaard, Phil. Trans. Roy. Soc. **A242**, 101 (1949).
- [13] E. Treffitz, A. Schlüter, K. Dettmar, and K. Jörgens, Z. Astrophys. **44**, 1 (1957).
- [14] A. W. Weiss, Astrophys. J. **138**, 1262 (1963).

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